Archives

Ref.TH.1529-CERN

O(n) RELATIONS FOR COUPLING CONSTANTS AND SPACE-TIME DIMENSIONS IN DUAL MODELS

P . H. Frampton
CERN -- Geneva

ABSTRACT

It is proposed that certain daughter trajectories arise as a consequence of a higher underlying O(n) symmetry, with $n > 3$. This suggestion is motivated by the dual resonance model, where such a pattern arises naturally from the existence of a critical space-time dimension. This is easily confirmed in the model (and provides a simple test for what is the critical dimension) by considering the amplitudes for spinless particles.

The results of πN phase shift analysis are discussed to give a speculative phenomenological estimate of the appropriate higher symmetry.

Ref.TH.1529-CERN
19 July 1972

In this article we discuss relations between coupling constants which are implied by the existence of a larger symmetry than rotational invariance and which have consequences for parent-daughter and daughter-daughter relative widths at each mass level. If the simplest description of a set of states at a given mass level is in terms of a single irreducible representation of $O(n)$, then when this is restricted to a three-dimensional space, it is reducible in terms of $O(3)$ to give a family of daughters with definite relative coupling strengths.

The motivation for suggesting such a property is coming from the dual resonance models, and the phenomenological study of daughter strengths could provide a useful input into the attempt to construct a better such model. We therefore first describe exactly how the phenomenon occurs in the known dual models of highest internal consistency, and then go on to the phenomenological evidence. The reader with no interest in dual models may skip all except the last part.

We outline from an appropriate viewpoint the classification of physical states (for the lowest excited levels) in the unit intercept Veneziano model. Since partition numbers are essential to this discussion, let us introduce immediately the symbols $p(x)$ and $T^d(N)$ through

$$(p(x))^d = \left[\prod_{r=1}^{\infty} (1-x^r)^{-1}\right]^d = \sum_{N=0}^{\infty} T^d(N)\, x^N \qquad (1)$$

In the Fock space spanned by the Lorentz harmonic operator oscillators $a_\mu^{(n)}$, $a_\mu^{(n)\dagger}$ the number of linearly independent states at the N^{th} level ($N = -\sum_{n=1}^{\infty} n a_n^+ \cdot a_n$) is $T^4(N)$, an appreciable number of which, corresponding to the coefficient of x^N in $[p^4(x)-p^3(x)p(x^2)]$ have negative norm (are ghosts) in view of the negative metric. Virasoro 1) conjectured that the gauge symmetry of the unit intercept case is sufficient to ensure decoupling of all but positive norm states on mass shell. The number of non-zero norm physical states at the N^{th} level is given by $T^3(N)-T^3(N-1)$. Explicit expressions for a smaller number, $T^2(N)$, of linearly independent positive norm states have been constructed in Ref. 2), and again in Ref. 3).

The Virasoro proposal 1) has recently been ingeniously proved 4) by a demonstration of the property that if one considers the model in $d = 26$ space-time dimensions (1 time + 25 space) there is an upper limit $T^{24}(N)$ on the number of non-zero norm physical states. Since either of the methods of Refs. 2) and I are generalizable to construct $T^{d-2}(N)$ positive norm physical states in d dimensions, it was immediately deduced 4) that there are no negative norm ghosts in the model for any $d \leq 26$, in particular for the value of $d = 4$.

Using the method of I in $d = 26$ dimensions, one obtains therefore <u>all</u> physical states as irreducible representations of $O(25)$. To establish notation we recall how finite dimensional irreducible representations of $O(n)$ are found in a tensor basis $^{5)}$. One defines an r^{th} rank tensor as one which transforms under the n by n orthogonal matrix a_{ij} according to

$$T'_{i_1' i_2' \dots i_r'} = a_{i_1 i_1'} a_{i_2 i_2'} \dots a_{i_r i_r'} T_{i_1 i_2 \dots i_r} \tag{2}$$

One can classify the tensor by an invariant indicial symmetry pattern, represented by a Young tableau with r boxes. For orthogonal transformations we make invariant contractions, whereupon the Young tableau may be restricted to those with no more than $\nu = [n/2]$ rows, where $[n/2]$ means the largest integer contained in $n/2$. Taking the rows to have lengths $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_\nu$ we may characterize the irreducible representation by the sequence of numbers $\{\lambda_1 \lambda_2 \lambda_3 \dots \lambda_\nu\}$ where $\sum_{i=1} \lambda_i = r$. For $O(3)$, $\nu = 1$ and we characterize by one number $\{\lambda\}$, which can be identified with the angular momentum J. For $O(25)$ there are twelve parameters to characterize an irreducible representation which we denote by $\{\lambda_1 \lambda_2 \lambda_3 \dots \lambda_{12}\}$.

We apply now the method of I to construct physical states for a general d dimensional space-time, as irreducible representations of $O(d-1)$. Starting with the vacuum state $|0\rangle$ we build up the spurious and physical states at each level N by using as raising operators the spatial part of the Virasoro gauge operators

$$\mathcal{L}_n = \frac{1}{2}\sum_{r=1}^{n-1}\sqrt{r(n-r)}\;\underline{a}^{(r)\dagger}\cdot\underline{a}^{(n-r)\dagger} + \sum_{r=1}^{\infty}\sqrt{r(n+r)}\;\underline{a}^{(r+n)\dagger}\cdot\underline{a}^{(r)} \tag{3}$$

with $n = 1, 2, 3, \dots$ and $\underline{A}\cdot\underline{B} = \sum_{i=1}^{d-1} A_i B_i$. The results are, for the physical states at the first five levels

$N = 0$ $\qquad |0\rangle$

Young tableau • or $\{00000\dots\}$

$N = 1$ $\qquad a_i^{(1)\dagger}|0\rangle \qquad i = 2, 3, 4, \dots\dots, (d-1).$

Young tableau □ or $\{10000\dots\}$

[We have chosen the 1-axis as longitudinal for this massless state.]

N = 2

$$\left[a_i^{(1)+} a_j^{(1)+} - \frac{\delta_{ij}}{(d-1)} \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \right] |0\rangle$$

Young tableau $\begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array}$ or $\{20000\ldots\}$

N = 3

$$\left[a_i^{(1)+} a_j^{(1)+} a_k^{(1)+} - \frac{1}{(d+1)} \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \left(\delta_{ij} a_k^{(1)+} + \delta_{jk} a_i^{(1)+} + \delta_{ki} a_j^{(1)+} \right) \right] |0\rangle$$

Young tableau $\begin{array}{|c|c|c|}\hline \ & \ & \ \\ \hline\end{array}$ or $\{30000\ldots\}$

$$\left(a_i^{(1)+} a_j^{(2)+} - a_j^{(1)+} a_i^{(2)+} \right) |0\rangle$$

Young tableau $\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}$ or $\{11000\ldots\}$

N = 4

$$\begin{aligned} \Big[& a_i^{(1)+} a_j^{(1)+} a_k^{(1)+} a_l^{(1)+} - \\ & - \frac{1}{(d+3)} \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \Big(\delta_{ij} a_k^{(1)+} a_l^{(1)+} + \delta_{ik} a_j^{(1)+} a_l^{(1)+} + \delta_{il} a_j^{(1)+} a_k^{(1)+} + \\ & \qquad + \delta_{jk} a_i^{(1)+} a_l^{(1)+} + \delta_{jl} a_i^{(1)+} a_k^{(1)+} + \delta_{kl} a_i^{(1)+} a_j^{(1)+} \Big) \\ & + \frac{1}{(d+1)(d+3)} \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \left(\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right) \Big] |0\rangle \end{aligned}$$

Young tableau $\begin{array}{|c|c|c|c|}\hline \ & \ & \ & \ \\ \hline\end{array}$ or $\{40000\ldots\}$

$$\begin{aligned} \Big[& 2 a_i^{(1)+} a_j^{(1)+} a_k^{(2)+} + \left(a_i^{(2)+} a_j^{(1)+} + a_i^{(1)+} a_j^{(2)+} \right) a_k^{(1)+} - \\ & - \frac{1}{(d-2)} \Big\{ \left(\underline{a}^{(1)+} \cdot \underline{a}^{(1)+} \right) \left(2 \delta_{ij} a_k^{(2)+} - \delta_{ik} a_j^{(2)+} - \delta_{jk} a_i^{(2)+} \right) \\ & \qquad - \left(\underline{a}^{(1)+} \cdot \underline{a}^{(2)+} \right) \left(2 \delta_{ij} a_k^{(1)+} - \delta_{ik} a_j^{(1)+} - \delta_{jk} a_i^{(1)+} \right) \Big\} \Big] |0\rangle \end{aligned}$$

Young tableau $\begin{array}{|c|c|}\hline \ & \ \\ \hline \ & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array}$ or $\{21000\ldots\}$

$$\left[\frac{1}{2} \left(a_i^{(1)+} a_j^{(3)+} + a_j^{(1)+} a_i^{(3)+} \right) - \frac{\delta_{ij}}{(d-1)} \left(\underline{a}^{(1)+} \cdot \underline{a}^{(3)+} \right) \right] |0\rangle$$

Young tableau $\begin{array}{|c|c|}\hline \ & \ \\ \hline\end{array}$ or $\{20000\ldots\}$

$\left(\underline{a}^{(1)+}\cdot\underline{a}^{(3)+}\right)|0\rangle$

Young tableau · or $\{00000\ldots\}$

This classifies all states obtained for the first five levels, by the method of I. For the lowest levels there is a remarkable absence of daughters in the critical dimension, i.e., the exponential growth begins <u>later</u>. One can check, by using Young tableau dimensionality formulae 5), that for an arbitrary d the number of components in each level from $N = 0$ to 4 sums up to $T^{d-2}(N)$, as expected *).

The result of Ref. 4) implies that this set of physical states forms a <u>complete</u> set, for $d = 26$, on the mass shell, i.e., we can insert in a residue at a pole $\alpha = N$ the completeness relation

$$\sum_{\lambda_N} |\lambda_N\rangle\langle\lambda_N| = 1 \qquad \text{(on mass shell)} \tag{4}$$

where the sum is over the $T^{24}(N)$ components of the $O(25)$ irreducible representations that we have classified. This completeness relation, Eq. (4), is true in general <u>only</u> for $d = 26$; to check this assertion, it is sufficient to consider explicitly only the level $N = 2$.

Because of the completeness of these states for $d = 26$, when we return to the realistic case, Refs. 6) and 7), of $d = 4$ space-time dimensions, by setting 22 components of momentum zero in the $d = 26$ model 4) (i.e., by restricting the representation space to a three-dimensional one), physical states of different spin, according to $O(3)$, have coupling constants related by the underlying $O(25)$ invariance. Where the dimension d enters the contraction coefficients, this will lead to a new (daughter) irreducible representation of $O(3)$. For example, the $\{20000\ldots.\}$ restricted representation of $O(25)$ at $N = 2$ becomes a sum of $\{2\}$ and $\{0\}$ irreducible representations of $O(3)$ with prescribed relative couplings. The physical consequence of such an $O(d-1)$ symmetry is then that it is impossible in any formation or production experiment to isolate a pure spin two f_0 resonance, without the associated

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*) We do not establish here any general rule for an arbitrary level N to decide directly (i.e., without building up from the lower levels) which irreducible representations of $O(25)$ are physical and which spurious. Such a rule is probably too complicated to be useful; it cannot depend only on the shape of the Young tableau, but must depend also on the partitioning of N into integers, since there exist Young tableaux (e.g., $\{2000\ldots.\}$ at $N = 4$) for which both a physical and a spurious state exist.

spin zero σ' contribution (the two contributions can be separated only by a subsequent partial wave analysis of the data). We can rewrite this tensor (now $i, j = 1,2,3$)

$$\Big[\underbrace{a_i^{(1)+}a_j^{(1)+} - \frac{1}{3}\delta_{ij}(\underline{a}^{(1)+}\cdot\underline{a}^{(1)+})}_{f_0} + \underbrace{\left(\frac{1}{3}-\frac{1}{25}\right)(\underline{a}^{(1)+}\cdot\underline{a}^{(1)+})}_{\sigma'}\Big]|0\rangle \qquad (5)$$

Similarly at $N = 3$ the $\{30000....\}$ $O(25)$ tensor becomes a sum of $\{3\}$ and $\{1\}$ in $O(3)$ with relative coefficient of the vector daughters prescribed as

$$\left(-\frac{1}{27}+\frac{1}{5}\right)(\underline{a}^{(1)+}\cdot\underline{a}^{(1)+}) \cdot \left(\delta_{ij}a_k^{(1)+} + \delta_{jk}a_i^{(1)+} + \delta_{ki}a_j^{(1)+}\right)|0\rangle \qquad (6)$$

The $\{11000....\}$ representation has a dimension-independent form and becomes the unnatural parity $\{1\}$ state at this level.

At $N = 4$ the totally symmetric $\{40000....\}$ becomes a sum of $\{4\}$ and $\{2\}$ (the expected scalar $\{0\}$ is already spurious); $\{21000....\}$ becomes a sum of $\{2\}$ and $\{0\}$; $\{20000....\}$ becomes $\{2\}$ plus $\{0\}$; and finally the fully contracted $\{00000....\}$ becomes $\{0\}$. To summarize at the level $N = 4$ the four $O(25)$ tensors become seven $O(3)$ tensors, but only four of their couplings are independent.

Of course all relative couplings within the model are prescribed uniquely from the beginning; what we are asserting here is only that, for factorization in an arbitrary multiparticle channel, the specification of one coupling constant appropriate to each $O(25)$ representation is sufficient to specify the coupling constants of all physical states for that channel. Let us confirm that this pattern of coupling constants exists by examining the simplest cases. Firstly the four-point function which can be written

$$\frac{\Gamma(-\alpha_s)\Gamma(-\alpha_t)}{\Gamma(-\alpha_s-\alpha_t)} = \sum_{n=0}^{\infty}\frac{R_n(t)}{n-\alpha_s}$$

with

$$R_n(t) = \frac{1}{n!}(\alpha_t+1)(\alpha_t+2).....(\alpha_t+n)$$

In terms of the scattering angle $z = \cos\theta_s$, we have, on mass shell, for unit intercept

$$R_2(z) = \frac{25}{8}\left[\left(z^2-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{25}\right)\right] \qquad (7)$$

$$R_3(z) = 27\left[\left(z^3 - \frac{3}{5}z\right) + 3\left(\frac{1}{5} - \frac{1}{27}\right) z\right] \tag{8}$$

so that the relative parent daughter couplings for natural parity agree with those of Eqs. (5) and (6) respectively. One can proceed to the multiparticle functions. One simple example, only, will be given here; more complicated factorizations can be handled similarly. We write

$$\begin{aligned} B_5 &= \int dx\, dy\; x^{-\alpha_1 - 1} (1-x)^{-\alpha_2 - 1} (1-y)^{-\alpha_3 - 1}\, y^{-\alpha_4 - 1} (1-xy)^{-\alpha_5 + \alpha_2 + \alpha_3} \\ &= \sum_{m=0}^{\infty} \frac{R_m(\alpha_2 \alpha_3 \alpha_4 \alpha_5)}{m - \alpha_1} \end{aligned}$$

where, for example,

$$\begin{aligned} R_2 = {} & \frac{1}{2}(\alpha_5 - \alpha_2 - \alpha_3)(\alpha_5 - \alpha_2 - \alpha_3 + 1)\; B_4(-\alpha_3, 2 - \alpha_4) \\ & + (\alpha_5 - \alpha_2 - \alpha_3)(\alpha_2 + 1)\; B_4(-\alpha_3, 1 - \alpha_4) \\ & + \frac{1}{2}(\alpha_2 + 1)(\alpha_2 + 2)\; B_4(-\alpha_3, -\alpha_4) \end{aligned} \tag{9}$$

For comparison, we re-calculate R_2 according to the prescription

$$R_2 = \sum_{\lambda_2} \langle 0 | V(p_2) | \lambda_2 \rangle \langle \lambda_2 | V(p_3)\, D(s_{45})\, V(p_4) | 0 \rangle \tag{10}$$

where

$$V(p) = \exp\left(-\sqrt{2}\, p \cdot \sum \frac{a^{(r)+}}{\sqrt{r}}\right) \exp\left(\sqrt{2}\, p \cdot \sum \frac{a^{(r)}}{\sqrt{r}}\right)$$

$$D(s) = \int dy\; y^{-1 - \alpha_s + \sum_{r=1}^{\infty} r a^{(r)+} \cdot a^{(r)}}$$

and $\sum_{\lambda_2} | \lambda_2 \rangle \langle \lambda_2 |$ is the completeness relation implied by Eqs. (4) and (5). Expanding the vertex

$$V(p_3) = \left(1 - \sqrt{2}\, p_3 \cdot a^{(1)+} + (p_3 \cdot a^{(1)+})^2 + \dots\right) \exp\left(\sqrt{2}\, p_3 \cdot \sum_{r=1}^{\infty} \frac{a^{(r)}}{\sqrt{r}}\right) \tag{11}$$

one finds that these three terms (all other terms vanish) become respectively the three terms for R_2 in Eq. (9), for unit intercept.

The method of I is easily extended to the Neveu-Schwarz model 8). In place of $\mathcal{L}_n$ we use the raising operators

$$\mathcal{G}_n = \sum_{r=1}^{n-1/2} \sqrt{r}\, \underline{a}^{(r)+} \cdot \underline{b}^{(n-r)+} + \sum_{r=n+1/2}^{\infty} \sqrt{r}\, \underline{a}^{(r)+} \cdot \underline{b}^{(r-n)} + \\ + \sum_{r=1}^{\infty} \sqrt{r}\, \underline{a}^{(r)} \cdot \underline{b}^{(r+n)+} \qquad (12)$$

for $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots,$ to build up irreducible representations of the Neveu-Schwarz algebra. It is convenient to use the Fock space $\mathcal{F}_2$ of Ref. 9); we can then build up physical and spurious states from the pion state $|0\rangle$ $(N = \frac{1}{2})$, and find the following results for the physical states in a general d dimensional space time

$N = \frac{1}{2}$ $\qquad |0\rangle$

Young tableau $\cdot$ or $\{000\ldots\}$

$N = 1$ $\qquad b_i^{(\frac{1}{2})+} |0\rangle \qquad i = 2, 3, 4, \ldots\ldots, (d-1).$

Young tableau $\begin{array}{|c|}\hline \\ \hline\end{array}$ or $\{100\ldots\}$

$N = \frac{3}{2}$ $\qquad b_i^{(\frac{1}{2})+} b_j^{(\frac{1}{2})+} |0\rangle$

Young tableau $\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$ or $\{110\ldots\}$

$N = 2$ $\qquad b_i^{(\frac{1}{2})+} b_j^{(\frac{1}{2})+} b_k^{(\frac{1}{2})+} |0\rangle$ (totally antisymmetrised)

Young tableau $\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline\end{array}$ or $\{1110\ldots\}$

$$\left[\frac{1}{2} \left(b_i^{(\frac{1}{2})+} a_j^{(1)+} + b_j^{(\frac{1}{2})+} a_i^{(1)+} \right) - \frac{\delta_{ij}}{(d-1)} \left(\underline{a}^{(1)+} \cdot \underline{b}^{(\frac{1}{2})+} \right) \right] |0\rangle$$

Young tableau $\begin{array}{|c|c|}\hline & \\ \hline\end{array}$ or $\{2000\ldots\}$.

According to Refs. 4), 10), these irreducible representations form a complete basis of the physical states on mass shell when $d = 10$; as discussed already for the conventional model, the $O(9)$ representations restricted to a three-dimensional representation space become reducible representations of $O(3)$

which decompose into irreducible representations with related coupling constants. For $d = 4$, the representations listed above become [cf., Ref. 9] the π $(N=\frac{1}{2})$, the ρ $(N=1)$, the ω $(N=\frac{3}{2})$; at $N = 2$ the $\{1110...\}$ representation becomes the $\{0\}$ representation of $O(3)$ corresponding to the η' and the $\{2000....\}$ becomes a sum of $\{2\}$ (f_0) and $\{0\}$ (σ') with relative coefficient given by

$$\Big[\underbrace{\frac{1}{2}\left(b_i^{(\frac{1}{2})+} a_j^{(1)+} + b_j^{(\frac{1}{2})+} a_i^{(1)+}\right) - \frac{\delta_{ij}}{3}\left(\underline{a}^{(1)+} \cdot \underline{b}^{(\frac{1}{2})+}\right)}_{f_0} + \underbrace{+\left(\frac{1}{3} - \frac{1}{9}\right) \delta_{ij} \left(\underline{a}^{(1)+} \cdot \underline{b}^{(\frac{1}{2})+}\right)}_{\sigma'}\Big] |0\rangle \tag{13}$$

To check this relation between coupling constants we consider the Lovelace formula $^{11)}$ and its pole expansion

$$- \frac{\Gamma(1-\alpha_s)\,\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)} = \sum_{n=1}^{\infty} \frac{R_n(t)}{n-\alpha_s} \tag{14}$$

and using the Neveu-Schwarz masses $m_\pi^2 = -\frac{1}{2}$, $m_\rho^2 = 0$, one finds on mass shell in terms of the scattering angle $z = (\cos\theta_s)$

$$\begin{aligned} R_2 &= \alpha_t(\alpha_t+1) \\ &= \frac{9}{4}\left[\left(z^2-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{9}\right)\right] \end{aligned} \tag{15}$$

as expected from the $O(9)$ invariance exhibited in Eq. (13).

Thus one can now realize that the underlying $O(25)$ invariance of the conventional model, and the $O(9)$ invariance of the Neveu-Schwarz model is not so deeply hidden when one simply studies the $\alpha(s) = 2$ residue of the beta function, and the Lovelace formula, respectively. This provides a rather simple method for obtaining the critical dimension of a model, once the amplitude for four scalar particles is known.

We now discuss two alternative possibilities.

One possibility, the more aesthetic one, is that the "correct" dual model, if it exists, has critical dimension $d = 4$. Then all parent-daughter relations of the type discussed above will be lost. We have nothing to add about this possibility except to mention that it might receive encouragement from the fact that in the attractive Lovelace formula, Eq. (14), for physical masses $m_\pi^2 = 0$ and $m_\rho^2 = \frac{1}{2}$, at the level $\alpha(s) = 2$, the σ' decouples and the f meson parent may be recognized as a pure O(3) irreducible representation. Ghosts are absent in this four-point function for $d = 4$, but reappear for any $d \geq 5$.

The alternative possibility is that a realistic model should have $d > 4$, and it is this that we would like to explore, and possibly to advocate, here.

More important, quite independent and outside of the dual model framework, we may make an O(d-1) ansatz for the daughter strengths and examine what is d empirically.

First let us distinguish between d (the total number of components of the external momenta which can include the four Lorentz dimensions plus additional dimensions to accommodate internal quantum numbers) and a different dimension D which characterizes the degeneracy of the spectrum. From the experience of the models which are understood, we expect that for b sets of bosonic (commuting) oscillators and f sets of fermionic (anticommuting) oscillators the level degeneracy $\rho(N)$ of the level N is given by [*)]

$$\prod_{n=1}^{\infty} (1-x^n)^{-b(d-2)} \prod_{m=1}^{\infty} (1+x^{m-\frac{1}{2}})^{f(d-2)} = \sum_{N=0,\frac{1}{2},1,\ldots}^{\infty} \rho(N)\, x^N \tag{16}$$

and then the Hardy-Ramanujan asymptotic form of $\rho(N)$ is [12)]

$$\rho(N) \underset{N\to\infty}{\sim} A\, m^{-B}\, e^{m/T} \tag{17}$$

where $m \sim \sqrt{N}$ is the mass and T is the effective Hagedorn temperature [12)]. In terms of [**)]

$$D = (d-2)\,(b + \tfrac{1}{2} f) \tag{18}$$

[*)] In Eqs. (16) and (18) we have implicitly assumed that two dimensions of oscillators decouple in the critical dimension. If only one dimension were to decouple there (d-2) should be replaced by (d-1); none of the conclusions are altered.

[**)] Equation (18) assumes that the model is really in its critical dimension; for a model below its critical dimension (and assuming that then only one dimension is decoupled by gauge conditions - see previous footnote) then $3(b+\frac{1}{2}f) \leq D \leq (d-2)(b+\frac{1}{2}f)$. This does not alter the conclusion that few additional oscillators can be added in a "correct" dual model.

we have

$$B = \frac{1}{2}(D+1) \tag{19.a}$$

$$T = \frac{1}{2\pi}\sqrt{\frac{6}{\alpha' D}} \tag{19.b}$$

There are already some arguments to determine B and T of the level density, Eq. (17), and thereby D through Eqs. (19) :

1) Hagedorn 12) finds by fitting transverse momentum distributions in multiparticle production a temperature T corresponding to $D = 4, 5$ or 6.

2) Huang and Weinberg 13) have pointed out that in certain models of the early universe, no thermal equilibrium is possible unless $B \leq \frac{7}{2}$, which would imply $D \leq 6$.

3) Assuming the strong statistical bootstrap condition of Frautschi and Hamer 14) leads these authors to a level density where $B = 3$ corresponding to $D = 5$.

These three arguments therefore most favour the value $D \approx 5$.

To give an estimate of d we move on to the question of whether parent-daughter coupling relations of the kind discussed earlier in the article exist in Nature. Unfortunately, there is very scanty evidence about meson daughters, so we are forced to introduce uncertainties and to look at the baryon spectrum.

In the recent π N phase shift analysis of Almehed and Lovelace 15) there occur seven candidates for daughters spaced two units below a parent, two in the Δ sector and five in the N sector. These seven pairs are listed in the Table.

Following the discussion of O(n) irreducible representations given earlier, it is straightforward to find that for the decay widths of a spin J parent, and its related spin (J-2) daughter (into two ground states) are in the ratio

$$\frac{\Gamma_{J-2}}{\Gamma_J} = \frac{(2J^2 - 1/2)\ (d-4)}{(2J-1)\ (2J-5+d)} \tag{20}$$

Let us now make two strong simplifying assumptions : a) the O(d-1) relations for bosons can be taken over to the baryon sector directly, b) the parent-daughter width ratios are to be compared directly to total widths *) rather than to partial widths.

The use of these two assumptions leads to quite an interesting result. The estimated d is presented for the seven cases in the last column of the Table. We see that the values of d are surprisingly consistent and favour $d = 6$ or 7.

When we abstract from the dual models the concept of $O(n)$ relationships and <u>postulate</u> directly that such relationships hold for daughter resonances, then the πN phase shifts imply that the relevant group is most likely to be $O(5)$ or $O(6)$.

Taken together with the arguments based on the level density, we see that for dual models $D \approx (d-2) \approx 5$ and thence, from Eq. (18), either $b = 1$, $f = 0$ (as in the conventional model) or $b = 0$, $f = 2$. This suggests that one may not add a large number of new types of excitation (i.e., new oscillators) to form a realistic dual model. On the other hand, the extra $(d-4)$ dimensions may be essential to incorporate internal quantum numbers **).

More generally, we can see from relations of the type Eq. (20) that daughter widths increase rapidly as d increases above $d = 4$. This applies, of course, only to those daughters ("trace daughters") which arise from the reducibility of the $O(d-1)$ irreducible representations restricted to a three-dimensional space; such daughters constitute, however, the large majority of daughters in the model since their level degeneracy increases exponentially with a higher exponent than the corresponding degeneracy of all other daughters (i.e., "genuine daughters" which are present already in the critical dimension) ***).

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*) Note that Eq. (20) does <u>not</u> contain a threshold dependence k^4, since it is the <u>reduced</u> coupling constants $(g^2 k^{2\ell})$ which are related directly in our reduction $O(d-1) \rightarrow O(3)$. In terms of the more conventional coupling constants, this means that $(g_J^2/g_{J-2}^2) \rightarrow \infty$ as $k \rightarrow 0$ such that the ratio of the widths (<u>not</u> the reduced widths) remains constant.

**) It is possible to make plausible that the Pomeron can remain a pole if the integral over a component of loop momentum is replaced by a sum over an infinite number of values of a discrete quantum number.

***) At intermediate energies the low partial waves (especially those for $\ell < \sqrt{s}$) in the beta function are too strong when compared to phenomenological fits [16]; in other words these daughter resonances are on average too wide. We may attribute this <u>in part</u> to the fact that the model has a critical dimension $d = 26$ which is such that $d \gg 4$ (or 7). The trace daughters decouple in the critical dimension, become negative width ghosts for $d > 26$ and develop progressively larger positive widths as we decrease d below the critical dimension.
In any event, further and more important absorption of these low partial waves is of course expected when Regge cuts are introduced in unitarity corrections.

It will be interesting if further empirical evidence for channel-independent parent-daughter strengths, as vestiges of a higher dimensionality, can be accumulated.

It is a pleasure to thank members of the Theoretical Study Division for interesting discussions, in particular mentioning H.B. Nielsen, M. Toller and G.C. Wick.

TABLE

PARENT RESONANCE	DAUGHTER RESONANCE	PARENT WIDTH (MeV)	DAUGHTER WIDTH (MeV)	(d-1)
F_{37} (1925)	P_{33} (2150)	200	200	5.0
F_{35} (1875)	P_{31} (1900)	250	200	4.46
G_{17} (2225)	D_{13} (2075)	150	150	5.0
F_{17} (2000)	P_{13} (1850)	200	300	6.6
F_{15} (1688)	P_{11} (1720)	140	160	5.46
D_{15} (2100)	S_{11} (2100)	150	200	6.2
D_{15} (1683)	S_{11} (1670)	150	120	4.46

Mean (d-1) = 5.3

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