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UNITARITY IN DUAL RESONANCE MODELS

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A B S T R A C T

We discuss some elementary consequences of the fact that all excited levels in dual resonance models are fundamentally unstable. Our results are relevant in order to understand the role of unitarity in the framework of these schemes.

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The dual resonance model presently under study [1] is an ansatz for the scattering amplitude of any number of scalar particles which satisfies most of the general properties one would like to associate to the S matrix of strong interactions. As is well known, the most apparent shortcoming of this model is its lack of unitarity, a property which is necessarily lost in the limit of infinitely narrow resonances. We shall not discuss problems connected to the fact that some of the excited states of the theory have imaginary coupling ("ghost" states). A compensation mechanism for these states has been found [2] and it is not impossible that dual amplitudes can be constructed which are completely "ghost" free. In the following we shall disregard the complications due to the possible presence of those unphysical states.

Attempts have already been made by several groups [3],[4] in order to build up a unitarization procedure in this framework. The most ambitious of such programmes is based on the idea [4] that the starting tree approximation is some kind of a Born term and that unitarity is automatically enforced in a perturbation manner by computing higher order loop contributions [5]. More precisely, if we associate the coupling constant f with each vertex, the n point amplitude will be of order f^{n-2}. The sum of all possible loops will give rise to a power expansion in f in which unitarity is satisfied consistently at each order. At any finite order, unitarity will hold in the Hilbert space of many excited (stable and unstable) states, but, at the end, unitarity in the complete space of all stable (asymptotic) states will, hopefully, be recovered. Conventional field theory seems to support this point of view.

There is, however, an important difference between the dual models and conventional field theory. Indeed, in our case all excited states are fundamentally unstable. As a consequence, many experimental quantities cannot be estimated without introducing a width right from the beginning. A general treatment of perturbation graphs with unstable particles has been given by Veltman in the framework of conventional field theory [*]. It will be very interesting to extend this treatment

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*) We thank Professors M. Veltman [6] and L. Van Hove for having illustrated some of the important aspects of this problem.

to dual resonance models. Waiting for a completely general treatment of the problem, we want to make some elementary remarks which exhibit some of the new features coming from unstability of the excited levels.

The situation is best illustrated in the following example. Consider the production amplitude $a+b \to c+d+e$ of Fig. 1. The corresponding five-point function has pole-type singularities in the final subenergies, e.g.,

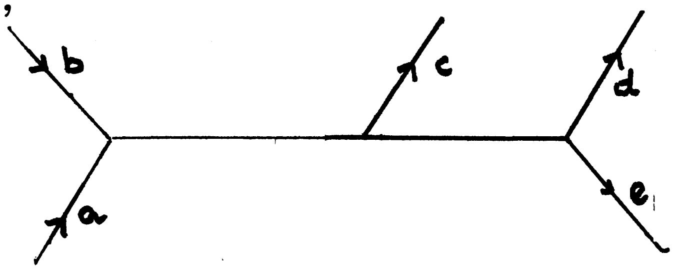

Fig. 1

in $(P_d+P_e)^2$, corresponding to the squared mass of some excited level. If the incoming energy is not too low, some of those poles will be in the physical region and therefore, when integrating over the final momenta in order to get the cross-section, one will find an integral of the form :

$$I \propto \int d(p_d+p_e)^2 \left| \frac{1}{(p_d+p_e)^2 - m_R^2 + i\Gamma_R} \right|^2 = \int ds \frac{1}{(s-m_R^2)^2+\Gamma_R^2} \simeq \frac{1}{\Gamma_R} \xrightarrow[\Gamma_R \to 0]{} \infty \tag{1}$$

Equation (1) shows that the cross-section diverges as $1/\Gamma_R$ when $\Gamma_R \to 0$; in particular, the integral does not exist if we use directly the unmodified five-point function *). Of course, the physical way to get around this difficulty consists in giving a finite total width to each unstable level. Such a width will be related through unitarity to the total decay rate of the unstable level. In other words, the width will depend again on the coupling constant f. This means that the cross-section for the process under consideration will have the form

*) In the V treatment this infinity is seen to cancel against similar terms appearing in some higher connection to two-particle production.

$$\sigma = f^6 I(f) \tag{2}$$

where the integral I depends on f through the widths of the resonances and $I(f)_{\overrightarrow{f\to 0}}\infty$. The presence of a non-zero width has thus as a consequence that the cross-section does not have a simple power (f^6) dependence on f. In this letter we want to examine some elementary consequences of this new feature coming from the presence of unstable particles. The outcome is, in our opinion, both trivial and interesting.

Consider first the calculation of a total cross-section $\sigma(a+b \to \text{anything})$. We wish to stress that for us only the stable multiparticle final states can appear in the definition of "anything". For a definite multiparticle final state (which we denote by X) the scattering amplitude will have contributions of the form (see Fig. 2) $a+b \to \text{resonance} \to X$. These are not the only contributions; however, as we shall see later, other terms although quite interesting will lead to final results of higher order in f.

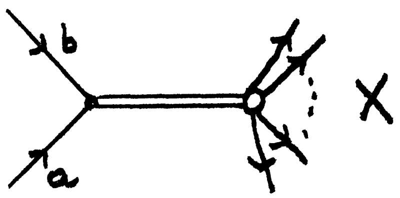

Fig. 2

In a strict narrow resonance approximation, we can write for the scattering amplitude

$$T_{ab\to X} = \sum_{n,r} \gamma^{ab}_{n,r} \frac{1}{s - s_n} \gamma^{X}_{n,r} \tag{3}$$

where the label r denotes the different states of mass$^2 = s_n$. Again, in order to compute a finite cross-section, we have to assign a finite width to each resonance. Choosing as intermediate states the eigenstates of the mass matrix, we can rewrite the correct formula as

$$T_{ab\to x} = \sum_{n,r} \gamma^{ab}_{n,r} \frac{1}{s - s_{n,r} - i\Gamma_{n,r}} \gamma^{x}_{n,r} \tag{4}$$

The total cross-section will then be

$$\sigma_{Tot} = \sum_{x} |T_{ab\to x}|^2 = \sum_{n,r} \gamma^{ab}_{n,r} \frac{\sum_x |\gamma^x_{n,r}|^2}{(s - s_{n,r})^2 + \Gamma^2_{n,r}} \gamma^{ab\,*}_{n,r} \tag{5}$$

However, because of unitarity

$$\Gamma_{n,r} = \sum_{x} |\gamma^{x}_{n,r}|^2 \tag{6}$$

so that finally we find

$$\sigma_{Tot} = \sum_{n,r} |\gamma^{ab}_{n,r}|^2 \frac{\Gamma_{n,r}}{(s - s_{n,r})^2 + \Gamma^2_{n,r}} \simeq \sum_{n,r} |\gamma^{ab}_{n,r}|^2 \delta(s - s_n) \tag{7}$$

The second equality does not hold in a strict local sense but only after integration of the two sides in s over regions large with respect to Γ. If we now recall the form of $T(ab\to ab)$, factorized in its s channel resonances, we realize immediately that

$$\mathrm{Im}\, T_{ab\to ab} = \sum_{n,r} (\gamma^{ab}_{n,r})^2 \delta(s - s_n) \simeq \sigma_{Tot} \tag{8}$$

The physical meaning of Eq. (7) is clear in the limit of narrow resonances. Once a narrow resonance is created, the final outcome of it is of no interest for the total cross-section which is simply given by the integrated probability of formation of any "compound" excited particle. This result was therefore to be expected on physical grounds. What is more surprising is Eq. (8), which states that the optical theorem applied to the "non-unitarized" four-point function already leads to a reasonable estimate of the total cross-section (as far as counting of orders in f^2 goes).

We stress again, however, that due to the huge number of states in the theory, the second equality in Eq. (7) will certainly not hold locally but only in the integrated sense explained above, especially at high energy. A more satisfactory way of writing Eq. (8) will then be :

$$\sigma_{Tot} = \left[\mathrm{Im}\, T_{ab \to ab} \right]_{average} \tag{9}$$

It is well known that such an average is just given by a Regge formula, so that, to lowest order in f^2, the forward Regge term gives the total cross-section at high energy (i.e., satisfies the optical theorem).

The result given in Eqs. (8) and (9) can be generalized in a straightforward manner once we realize that the initial and final states (which in our example were both equal to $a+b$) do not play any special role. We find therefore to lowest order in the coupling constant

$$\sum_n \langle A | T^+ | n \rangle \langle n | T | B \rangle = \left[\mathrm{Im} \langle A | T | B \rangle \right]_{average} \tag{10}$$

where $|n\rangle$ are the stable intermediate states.

Let us make now a few observations about our results :

1) Equation (10) shows that the dual multiparticle amplitudes without loops, if correctly interpreted, contains already in it important contributions from unitarity : to lowest order in f^2 it is already "unitary" ! We shall discuss at the end what the main higher order corrections will be. We also remark that this approximate unitarity is obtained from very general properties of the dual resonance model, i.e., from the fact that the general amplitude factorizes completely [7] and that resonance widths are supposedly small. It is not hard to realize that, given a definite resonance model, all widths can be calculated directly in terms of f^2 and therefore the assumption of small widths can be in principle checked a posteriori.

2) Arguments based on power counting in the coupling constant in analogy with conventional perturbation theory may lead to misleading results. As an example, the leading contribution to the total cross-section for production of n particles is of order f^2 whereas conventional power counting would give f^{2n}. As a consequence, the physical basis of the loop approach to unitarity (which is essentially based on such perturbative power counting) should be carefully re-examined.

We now add a few remarks on higher order corrections in the coupling constant. We expect two kinds of such corrections. The first will be to give rise to higher effects in mass shifts and widths of resonances. Those effects will be responsible for the details of the averaging process and will be probably rather hard to compute.

The second, more important effect, will come when we consider those components of the dual amplitudes in which the initial and final states $|A\rangle$ and $|B\rangle$ of Eq. (10) cannot produce a single resonance. As an example, consider again the case of elastic two-body scattering $|A\rangle = |B\rangle = |a+b\rangle$. There are contributions to the total cross-section coming from terms like the one of Fig. 3, where the twist (denoted by a cross) does not allow the two resonances R_1 and R_2 to collapse into

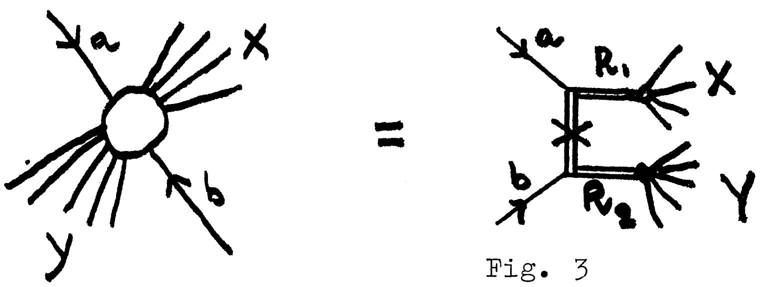

Fig. 3

a single one. By the arguments given before the lowest order contribution of this diagram to the total cross-section (of order f^4) is given by the s channel imaginary part of the non-planar loop of Fig. 4.

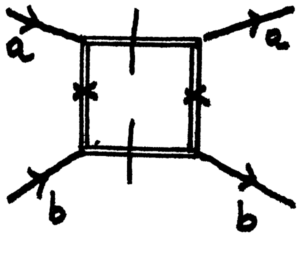

Fig. 4

Besides other effects, this contribution to the total cross-section hopefully will give diffractive effects (Pomeron exchange) in agreement with the Harari-Freund point of view [8]. Actually a number of authors [9] have already considered this diagram to be associated with the Pomeranchuk singularity.

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